

1 Balancing procedure for basic state

Basic state as normally defined by the climatological mean fields sometimes contains an artificially large divergent tendency which is not found in the GCM. This comes from the climatological mean not necessarily satisfying the governing equations. This large divergent tendency may bring an undesired effect in computing a residual forcing for the nonlinear computation (section 6) and even in solving steady linear sponse. To avoid this, a balancing pocedure, or simply *initialization*, can be applied to the basic state, following Hoskins and Simmons (1975, QJRMS) and Hall (2000, JAS).

The essence of initialization is to minimize the divergent tendency by modifying T and π fields with fixed ζ , or modifying ζ with fixed T and π . Here we take the former way.

In the nonlinear divergent equation

$$\frac{\partial D}{\partial t} = \mathcal{D} - \nabla^2(\Phi + R\bar{T}\pi) \quad (1)$$

where $\mathcal{D}$ is the non-gravity wave part which consists of the advection and Laplacian of kinetic energy whereas the second term of the rhs is the source of gravity waves. Φ is the geopotential, and $\bar{T}(\sigma)$ is the base temperature. Using the hydrostatic relation, (1) is rewritten to

$$\frac{\partial D}{\partial t} = \mathcal{D} - \nabla^2(\Phi_s + \mathbf{W}T + R\bar{T}\pi) \quad (2)$$

where Φ_s is surface geopotential equivalent to the topography and $\mathbf{W}$ is the operator of the hydrostatic relation

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \vdots \\ \Phi_k \end{pmatrix} = \begin{pmatrix} c_p\alpha_1 & 0 & \dots & 0 \\ c_p(\alpha_1 + \beta_1) & c_p\alpha_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c_p(\alpha_1 + \beta_1) & c_p(\alpha_2 + \beta_2) & \dots & c_p\alpha_k \end{pmatrix} \begin{pmatrix} T_1 \\ T_1 \\ \vdots \\ T_k \end{pmatrix} \quad (3)$$

The balancing procedure requires zero tendency in (2), so that with a given $\mathcal{D}$, $\bar{T}$ and π T can be solved from

$$\nabla^{-2}\mathcal{D} = (\Phi_s + \mathbf{W}T + R\bar{T}\pi) \quad (4)$$

In practice, this calculation is carried out in the spectral wave space for each m and n . After T has been modified according to (4), a binomial filter is applied vertically in order to remove a 2-grid noise that has a large amplitude. Then a correction to π is computed such as to preserve the rhs of (4) approximately:

$$\Delta\pi = \frac{1}{K} \sum_k^K \frac{\Phi_k - \Phi_k^*}{R\bar{T}_k} \quad (5)$$

where Φ^* means the geopotential derived from the smoothed temperature. Since $\mathcal{D}$ depends also on π , the above procedure is iterated up to a condition that the rms of surface pressure becomes

smaller than a given threshold (set at 10^{-3} hPa) is satisfied. The iteration tends to converge very quickly, so that 5–10 iterations are sufficient for most cases.

Figures below are examples of the initialization effects to temperature and surface pressure.

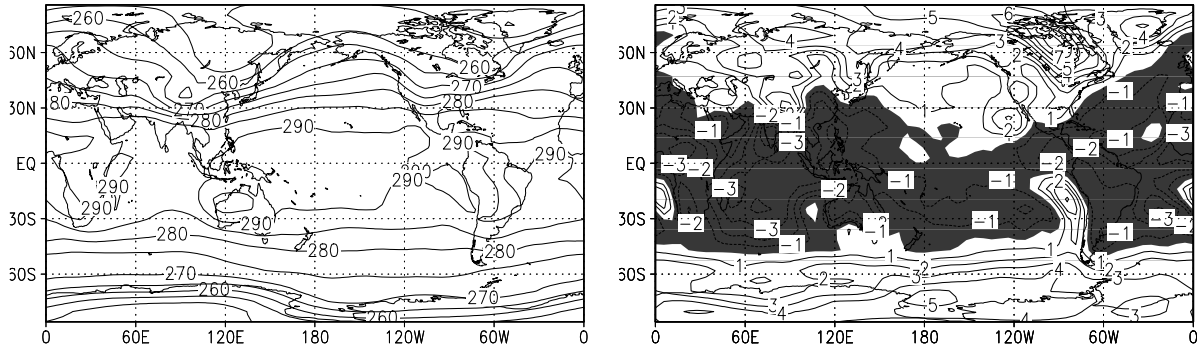


Figure 1 (left) Basic state temperature for T21L5 LBM at $\sigma = 1$ modified by the balancing procedure, and (right) difference between the original and modified temperature (latter minus former). The unit is K.

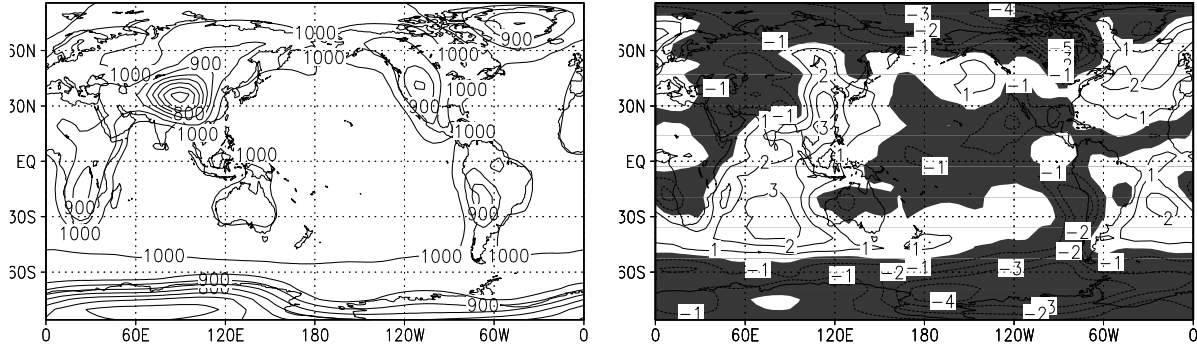


Figure 2 As in Fig. 1 but for surface pressure. The unit is hPa.