Note on a LSC scheme incorporated in the LBM2.1 (08/15/02)

Hiro Watanabe (hiro@ees.hokudai.ac.jp)

1 Original nonlinear scheme

Stratiform rainfall as often represetned by the large–scale condensation (LSC) process is parameterized as follows. First, let us assume that tendencies of temperature and water vapor mixing ratio due to LSC process are represented as a relax-form equation.

$$\frac{\partial T}{\partial t} \quad \cdots \quad = \frac{1}{\tau_l}(T^* - T) \tag{1}$$

$$\frac{\partial q}{\partial t} \quad \cdots \quad = \frac{1}{\tau_l}(q^* - q) \tag{2}$$

where τ_l is the relaxation timescale of the LSC, equivalent to the lifetime of stratiform cloud, while T^*, q^* are reference temperature and specific humidity defined as

$$T^* = \alpha\tilde{T} + (1-\alpha)T \tag{3}$$

$$q^* = \alpha\tilde{q} + (1-\alpha)q \tag{4}$$

where α denotes a distribution coefficient which can express an efficiency of the LSC adjustment such that the adjustment becomes smaller for the grid only slightly saturated

$$\alpha = \frac{1}{2}\tanh(x) + \frac{1}{2} \qquad , \quad x = \frac{q - q_s(T)}{\beta} \tag{5}$$

A constant β is ordinary set at 1×10^{-2} kg/kg. $\tilde{T}, \tilde{q}$ are the virtual temperature and humidity which correspond to the values at complete moist–adiabatic adjustment and can be solved with an iteration.

$$\tilde{T} = T + \frac{L}{c_p}\{q - \tilde{q_s}\} \tag{6}$$

$$\tilde{q} = q_s(\tilde{T}) \equiv \tilde{q_s} \tag{7}$$

From (1)–(7) it is clear that for the strict adjustment of $\tau_l \to 0$ $T = T^* = \tilde{T}$ and $q = q^* = \tilde{q}$ are accomplished with Enthalpy conserved.

$$c_pT + Lq = c_p\tilde{T} + L\tilde{q}$$

2 Linearized scheme

Suppose any variable can be decomposed as

$$A = \overline{A} + A'$$

then we deduce equations for perturbations (A'). Here we again assume that the basic state $\overline{A}$ is known.

Rewrite (6)–(7) as

$$\begin{aligned} \tilde{T}' &= T' + \frac{L}{c_p}\{q' - \gamma T'\} \\ &= \left(1 - \frac{\gamma L}{c_p}\right) T' + \frac{L}{c_p} q' \end{aligned} \tag{8}$$

$$\tilde{q}' = \gamma T' \tag{9}$$

where

$$\gamma = \frac{\partial q_s}{\partial T}\Big|_{T=\overline{T}} \tag{10}$$

Also (3)–(4), which have been linear, are modified for perturbation

$$T^{*'} = \alpha \tilde{T}' + (1-\alpha) T' \tag{11}$$

$$q^{*'} = \alpha \tilde{q}' + (1-\alpha) q' \tag{12}$$

Note that α is now a function of basic state.

$$\alpha = \frac{1}{2} \tanh\left\{\frac{\overline{q} - q_s(\overline{T})}{\beta}\right\} + \frac{1}{2} \tag{13}$$

Using (8)–(12), a set of perturbation equations is obtained as:

$$\begin{aligned} \frac{\partial T'}{\partial t} \quad \cdots \quad &= \frac{1}{\tau_l}(T^{*'} - T') \\ &= \frac{1}{\tau_l}\left\{\alpha \tilde{T}' + (1-\alpha) T' - T'\right\} \\ &= \frac{1}{\tau_l}(\alpha \tilde{T}' - \alpha T') \\ &= \frac{\alpha}{\tau_l}\left(\frac{L}{c_p} q' - \frac{\gamma L}{c_p} T'\right) \\ &= \frac{\tilde{\alpha}}{\tau_l}(q' - \gamma T') \end{aligned} \tag{14}$$

$$\begin{aligned} \frac{\partial q'}{\partial t} \quad \cdots \quad &= \frac{1}{\tau_l}(q^* - q) \\ &= \frac{1}{\tau_l}\{\alpha \tilde{q}' + (1-\alpha) q' - q'\} \\ &= \frac{1}{\tau_l}(\alpha \tilde{q}' - \alpha q') \\ &= \frac{\alpha}{\tau_l}(\gamma T' - q') \end{aligned} \tag{15}$$

where

$$\tilde{\alpha} = \frac{\alpha L}{c_p} \tag{16}$$

3 Finite difference

(14)–(15) are discretized by means of the leapfrog and implicit schemes. Let n be the time step (' is dropped for simplicity)

$$\frac{T^{n+1} - T^{n-1}}{2\Delta t} \quad \ldots \quad = \frac{\tilde{\alpha}}{\tau_l}(q^n - \gamma T^{n+1}) \tag{17}$$

$$\frac{q^{n+1} - q^{n-1}}{2\Delta t} \quad \ldots \quad = \frac{\alpha}{\tau_l}(\gamma T^n - q^{n+1}) \tag{18}$$

Coefficients of γ, α, and $\tilde{\alpha}$ are determined following (10), (13), (16), respectively, and all is the function of basic state independent of time. Solving (17), (18) yields

$$T^{n+1}|_{LSC} = \frac{T^{n-1} + 2\Delta t \tilde{\alpha} q^n / \tau_l}{1 + 2\Delta t \tilde{\alpha} \gamma / \tau_l} \tag{19}$$

$$q^{n+1}|_{LSC} = \frac{q^{n-1} + 2\Delta t \alpha \gamma T^n / \tau_l}{1 + 2\Delta t \alpha / \tau_l} \tag{20}$$

* Use the scheme differently for the moist STM and moist LBM. In the moist STM, may need to include additional linear damping over land similar to Zhang and Held's (1999) STM.

* The LSC scheme should be used with the cumulus scheme described by Watanabe and Jin (2003).

* When the model is used to reproduce the 'mean' state associated with the storm track, precipitation process may require a 'positive–definite' option to avoid negative rainfall.