Note on AIM in LBM and an 'artificial response' problem

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AIM has been developed and was successfully applied to the barotropic model. Here the linear baroclinic model (LBM) is used to solve forced steady solutions with AIM. Since most of the true LBM solutions cannot be obtained by the direct matrix inversion due to insufficient computer memory, it is not available to evaluate error ε. Instead, as in the barotropic model the norm ratio λ is used to evaluate convergence of the iteration. The norm ratio at iteration step *n* is defined as

$$\lambda^n \equiv \left\| \mathbf{X}^n - \mathbf{X}^{n-1} \right\| / \left\| \mathbf{X}^1 - \mathbf{X}^0 \right\|$$

where $\mathbf{X}^0$ is the initial guess. For simplicity, the energy norm is used to evaluate λ. Figure 1 shows the norm ratio for AIM and conventional time integration both applied to the T21L5 LBM. It is clear that the AIM converges faster than the time integration. Besides the basic characteristic toward the convergence appears to be determined by the operator $\mathbf{L_S}$ (compare black and red lines) although the convergence efficiency for sufficiently small λ (for example λ=1.e-3) is largely affected by $\mathbf{L_A}$, too.

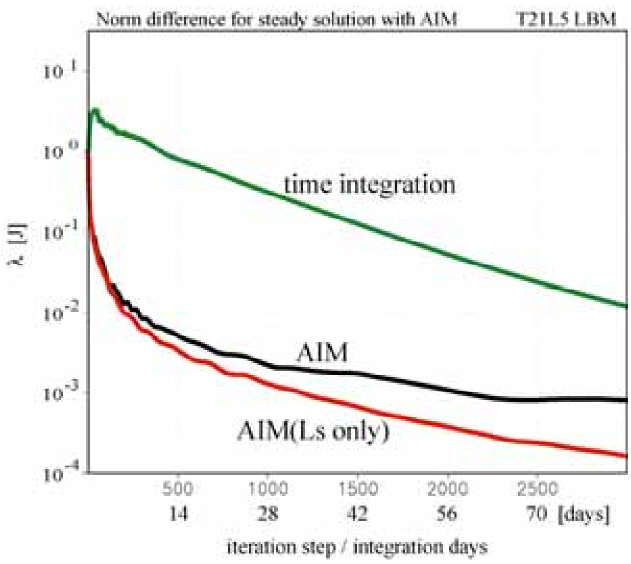

Figure 1: Energy norm differences for the T21L5 LBM responses obtained by the time integration (green) and AIM (black). The factor γ for AIM is set at 2000. The red line is for the AIM solution but with the zonally symmetric basic state.

Since the T21L5 resolution is coarse enough to compute the true (i.e. direct inversion) solution, the AIM solution in Fig. 1 at n=2071 at which λ becomes smaller than a threshold of 1.e-3 is compared with the true response (Fig. 2). Because of increasing numerical error and very slight difference in the implementation, the RMS error ε is yet around 0.1 (10%) at this convergence. But the response is sufficiently similar to each other, so that the λ <1.e-3 seems to be a reasonable threshold.

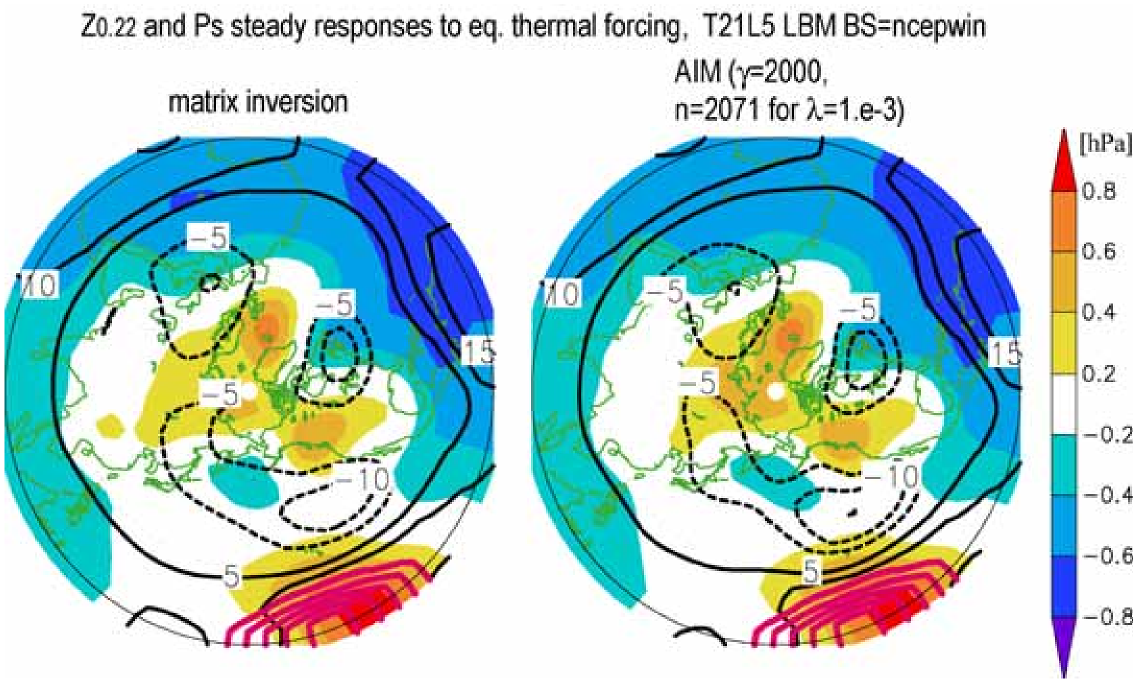

Figure 2: Steady response of σ=0.22 geopotential height (contour, interval 5m) and surface pressure (shade) to an equatorial thermal forcing (red thick contours) compued using the T21L5 LBM. (left) True response obtained by the matrix inversion, and (right) approximate solution using AIM.

The height response in Fig. 2 may not be very good to represent the PNA-like teleconnection, but this is mostly due to low vertical resolution. The same response pattern obtained in the T21L20 model (Fig. 3 left) is much improved. Note that discrepancies found between AIM and time integration in Fig. 3 are much larger than expected. The true solution at this resolution is not available, so that it is hard to identify whether AIM or time integration is closer to the true solution.

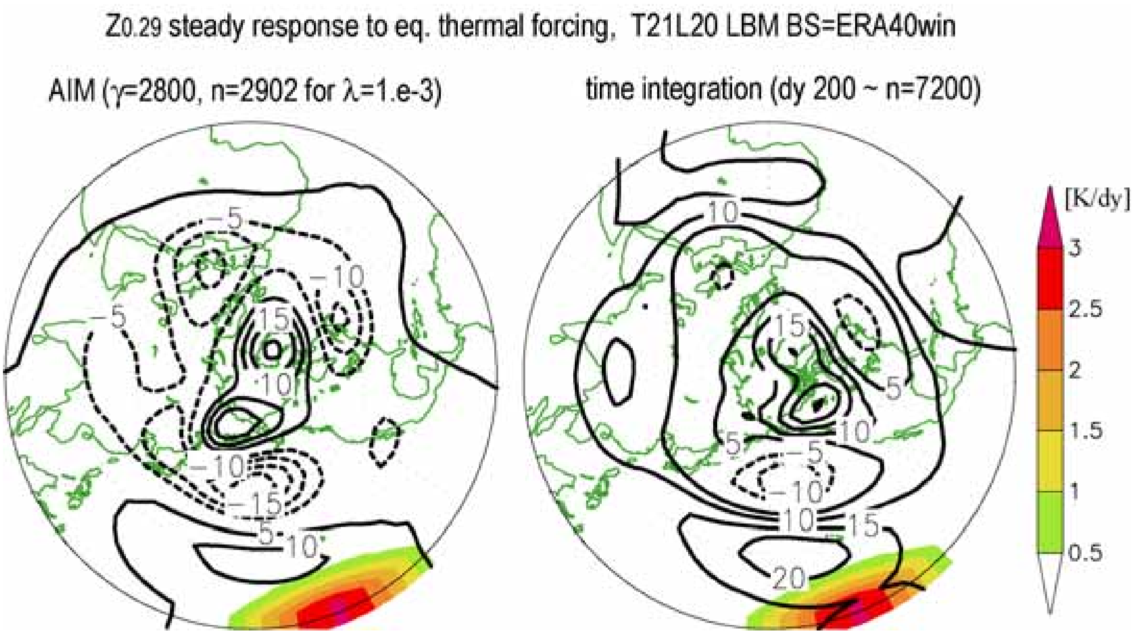

Figure 3: As in Fig. 2 but for T21L20 LBM. The height response at σ=0.29 (contour) is obtained with (left) AIM and (right) time integration.

It is interesting to compare the two solutions in Fig. 3 but with slightly unstable system. Namely, the model damping coefficients (0.5,0.5,0.5,20,20,…,20,1,1 days) are altered to (0.5,1,1,20,20,…,20,1,1 days), which bring the linear operator **L** marginally unstable. In Fig. 4 the height responses of AIM at n=1355 and n=5000 are shown, together with the time integration responses at roughly corresponding days (day 38 and 200). In both the computations, responses at smaller iteration (left panels) are not far from the stable steady solutions in Fig. 3, but those after sufficient iterations are quite different, governed by linearly unstable modes (right). The iteration of course never converges and explores soon after. In the time integration, there is no constraint of steadiness, so that the unstable baroclinic waves continuously propagate eastward and amplify. However, the AIM (and probably matrix inversion as well) does not allow such a propagation, so that the response may only amplify at particular area (i.e. the most baroclinically unstable region) without phase propagation, resulting in an artificially steady response near Siberia as has been sometimes found in the matrix inversion.

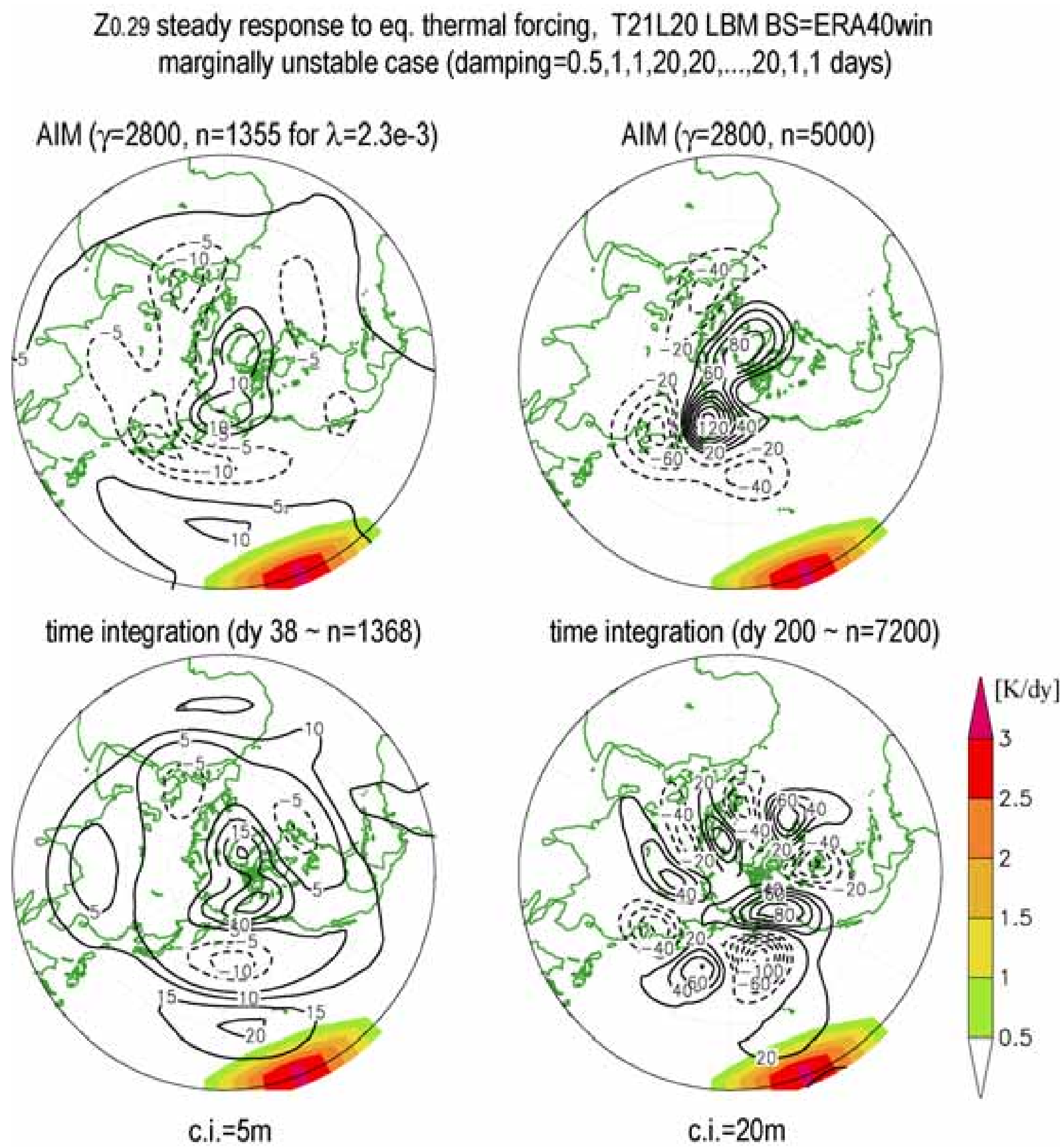

Figure 4: As in Fig. 3 but for the responses under marginally unstable condition. The height response at σ=0.29 (contour) is obtained with (top) AIM and (bottom) time integration. Both the computations never converge.